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The purpose of the research was to study the proceses by which stochastic motion is self-generated in deterministic systems, and the consequences of the resulting stochasticity.

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Final Report

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ONR Contract N00014-95-I-0361

RESEARCH ON SELF-GENERATED STOCHASTIC MOTION

1 October 1988 – 31 August 1996

I. Summary of Technical Accomplishments

The purpose of the research under this contract was to study the processes by which stochastic motion is self-generated in deterministic systems, and the consequences of the resulting stochasticity. We consider representative problems, each of which incorporates one or more of the main underlying physical phenomena. Phenomena that we have studied under this contract include diffusion through stochastic webs, diffusion along resonances in more than two degrees of freedom (Arnold diffusion), equipartition in oscillator chains, and the dynamics of coupled dissipative systems, such as phase locking, chaos, synchronization, and synchronized chaos.

1. *Diffusion Through Stochastic Webs*

Diffusion has been explored in a two-dimensional phase space in which a connected separatrix layer (web) of intrinsic stochasticity bounds regions of regular motion (tiles). In the presence of weak extrinsic noise, if the web diffusion dominates, the noise slows the web diffusion rate; if the extrinsic

diffusion dominates, the diffusion is enhanced. Analytic calculations agree well with numerical results (Pub. 1, 4).

2. *Arnold Diffusion*

When several standard maps are coupled together, KAM surfaces cannot isolate stochastic regions, and particles diffuse along stochastic layers by the process of Arnold diffusion. For the case of two coupled standard maps the rate of Arnold diffusion has been calculated both locally around a particular KAM curve and globally across many cells of the 2π periodic mapping. When more than two standard maps are coupled, the diffusion rate increases, depending on the total number of maps, N, and the number of phases in each coupling term, m, where $2 \leq m \leq N$. As N is increased, the diffusion rate increases as $N^{1/2}$, the length of the diagonal in the action space. As m is increased, the diffusion rate increases because the phase of the coupling term for a particular map becomes less correlated with the phase of the map itself. An analytic calculation of local diffusion for the m and K dependence has been developed, which is in good agreement with numerical results. The calculated local rate of diffusion and global phase space arguments are used to calculate the global diffusion (Pub. 7. 18).

3. *Equipartition in Coupled Oscillator Chains*

The energy transitions and time scales have been studied in the Fermi-Pasta-Ulam (FPU) oscillator chain and in a set of coupled pendula, for which the energy E, initially in a single or small group of low frequency modes, is distributed among modes. The energy transitions, with increasing energy,

have been classified. At low energy the linear parts of the energies are distributed in a geometrically decreasing series. A transition occurs such that above this transition there is strong local coupling among neighboring modes with a characteristic resonant frequency. There is a second transition at a critical energy for which stochasticity among low-frequency resonances transfers energy into high-frequency resonances by the Arnold diffusion mechanism. Above this transition we determine a universal scaling for the time scale to approach equipartition among the modes (Pub. 6, 16, 23, 24).

4. *Chaos and Synchronization in Coupled Phase-Locked Loops*

A broad study has been undertaken to study the chaos, synchronization, and synchronized chaos in coupled dissipative mappings. The device used for this exploration was digital phase-locked loops (DPLL's). The study also concerned the use of synchronized chaos in communications applications, in collaboration with industry. The basic work has also been supported with an AASERT grant administered by the ONR contract.

A single first order DPLL is topologically equivalent to a circle map, having the generic properties of phase-locking and chaos above a certain gain threshold. Coupled non-uniformly sampling DPLL's have interesting new dynamics which arise from a new class of coupled mappings. These have been studied theoretically, numerically, and experimentally. It was shown that chaotic signals can be synchronized, with a possible application to low-probability-of-intercept communication. Recent work has concerned theoretical and numerical studies of the behavior of many coupled loops, with

various coupling configurations, and the effect of noise on synchronization (Pub. 2, 5, 8, 12, 14, 15, 20, 21, 22, 27).

5. *Stochasticity in Toroidal Plasmas*

Various studies have been made of stochasticity and diffusion in toroidally confined plasmas. Some of the mechanisms that have been considered concern parametricly driven diffusion, resonance overlap, and large amplitude effects of instabilities. The results have supplied explanations for enhanced loss across edge plasmas and for the sawtooth oscillations in Tokamak discharges (Pub. 9, 10, 17, 18).

6. *Control of Chaos*

We have had two projects in this area. The first was a study of the control of a platoon of automated vehicles. We have shown that linear controllers lead to unacceptably long stopping distances under extreme conditions. A nonlinear controller can also be used to supply beneficial nonlinear forces. For example, a nonlinear controller can apply the largest brake force when a vehicle is about to collide with its preceding vehicle, which can greatly shorten the stopping distance for a large amplitude event. However, nonlinearities also introduce forces that can lead to chaotic behavior.

Recent work concerns controlling chaos using nonlinear feedback with delay. Nonlinear feedback results in a larger basin of attraction to the stabilized orbit than linear feedback. For the simple test mapping studied (the logistic map) the dimension of the system increases from 1 to 2 by introducing control. We show in the case involving memory, for a particular choice of the

relationship between the control parameters, that the superstable orbit can be recovered without reducing the parameter space that can be controlled (Pub. 25, 27).

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(27) P. Khoury, M. A. Lieberman, and A. J. Lichtenberg, "Degree of Synchronization of Noisy Maps on the Circle," *Phys. Rev. E*, **54,** 3377 (1996).

III. Graduate Students

G. M. Bernstein, PhD, 1989

C. G. Goedde, PhD, 1990

B. P. Wood, PhD, 1991

J. DeLuca, PhD, 1994

W. E. Wonchoba, PhD, 1995

P. Khoury, PhD, 1997 (Expected)

M. de Sousa Vieira (Post-Doctoral)